# Monod-Wyman-Changeux Model

Huarui Zhou on 2021/10

First, assume that protein X has $n$ binding sites for ligand F and protein X has two states, $R$ and $T$. The binding of different numbers of ligands to protein X leads to transitions between R and T states. We denote the protein X bound with $k$ $(0 \le k \le n)$ ligands in the two states as $R_k$ and $T_k$, respectively. The dissociation constants $K_R$ and $K_T$ are independent of how many ligands are bound to protein X. When no ligands are bound, we have the following equilibrium:

$$R_0 \underset{}{\overset{L}{\rightleftharpoons}} T_0$$

where

$$L = \frac{[T_0]}{[R_0]}.$$

Moreover,

$$R_{k+1} \overset{K_R}{\rightleftharpoons} R_k + F, \quad T_{k+1} \overset{K_T}{\rightleftharpoons} T_k + F, \quad 0 \le k \le n-1.$$

To find the expression of $K_R$ and $K_T$, we only need to know the concentration of $F$, $R_k$, $R_{k+1}$ and $T_k$, $T_{k+1}$. However, $R_k$ is a mixture of different configuretions and the actual participants in the dissociation reaction are the individual configurations. Therefore, we need to determine the concentration of each single configuration. Since $R_k$ has $C_n^k$ configurations, the concentration for each configuration in $R_k$ is

$$\frac{[R_k]}{C_n^k}.$$

therefore, we have

$$K_R = \frac{(C_n^k)^{-1}[R_k][F]}{(C_n^{k+1})^{-1}[R_{k+1}]} = \frac{n-k}{k+1} \cdot \frac{[R_k][F]}{[R_{k+1}]}, \quad K_T = \frac{(C_n^k)^{-1}[T_k][F]}{(C_n^{k+1})^{-1}[T_{k+1}]} = \frac{n-k}{k+1} \cdot \frac{[T_k][F]}{[T_{k+1}]}.$$

Define

$$c = \frac{K_R}{K_T}, \quad \alpha = \frac{[F]}{K_R},$$

Thus, we can calculate the proportion of binding sites occupied by F relative to all binding sites as:

$$\bar{Y}_F = \frac{\sum_{k=1}^n k([R_k] + [T_k])}{n(\sum_{k=0}^n [R_k] + T_k)} = \frac{Lc\alpha(1+c\alpha)^{n-1} + \alpha(1+\alpha)^{n-1}}{(1+\alpha)^n + L(1+c\alpha)^n},$$

and the proportion of protein X in the R state relative to the total amount of protein X is:

$$\bar{R} = \frac{\sum_{k=0}^n [R_k]}{\sum_{k=0}^n ([R_k] + [T_k])} = \frac{(1+\alpha)^n}{(1+\alpha)^n + L(1+c\alpha)^n}$$

# References


[1] Monod J, Wyman J and Changeux J-P. On the nature of allosteric transitions: A plausible model. Journal of Molecular Biology, 1965(12) : 88-118.